

INDIAN STATISTICAL INSTITUTE

CHENNAI CENTRE

M. STAT. I YEAR: 2016-17.

MID SEMESTRAL EXAMINATION

Date: 24 January 2017

Max. Score: 40

Duration: 10:15 - 11:30

- 1 Consider the population of 4 Zones of Chennai with the following data on no. of households (x) and population count (y):

Zone	U_1	U_2	U_3	U_4
x	200	180	240	400
y	1016	923	1214	2017

For a given pilot survey, it is desired to draw a sample of 2 units (Zones) by ppx and without replacement.

- Explain your selection method giving reference to the random numbers chosen. [2]
 - Write down Desraj's ordered estimators for $Y = \sum_{i=1}^4 Y_i$ and the pooled estimators for each order. $[3+1]+[3+1] = [8]$
 - From (b) calculate Lahiri-Murthy (Symmetrized Des Raj) estimator for Y . [4]
 - Obtain an unbiased estimate of $V(\hat{Y})$ using (b). [2]
2. a) Explaining your notation, write down the Horvitz-Thompson estimator for the population total $Y = \sum_{i=1}^N Y_i$. Denote this by \hat{Y}_{HT} . [2]
- Obtain an expression for $\text{Var}(\hat{Y}_{HT})$ as given by Horvitz-Thompson as well as by Sen-(Yates)-Grundy. $[3+5] = [8]$
 - Give one example where $\hat{V}(\hat{Y}_{HT})$ is non-negative. [2] [p.t.o.]

3(a) With the usual notation, show that $\sum_{j \neq i}^N \pi_{ij} = (n-1)\pi_i$ for fixed sample size (n) designs. What happens when $\pi(n) \neq n$ (fixed) if $\pi(n) > 0$.
 $[4+2]=[6]$

b) When does the equality occur in the relation $\pi_i + \pi_j - \pi_{ij} \leq 1$.
 $[2]$

c). If $\pi(n)$ is the effective sample size (i.e. number of distinct units in n) and $E(\pi(n)) = \pi = [\pi] + f$, $0 \leq f < 1$, the fractional part, then show that $V(\pi(n)) \geq f(1-f)$.
 $[4]$